LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

SECOND SEMESTER – **APRIL 2015**

ST 2962 - MODERN PROBABILITY THEORY

Date : 25/04/2015 Dept. No. Max. : 100 Marks
Time : 01:00-04:00

SECTION A

Answer **ALL** of the following. **(10x2=20)**

1. Explain Probability space.
2. Define: Monotone Field.
3. What is convergence of random variables?
4. Define: Signed Measure
5. Define Mixture of Distributions.
6. Let X be a continuous Gamma Variate, derive, $M_x(\theta)$.
7. Derive the Harmonic Mean of Beta Distribution of first kind.
8. Explain equivalent convergence.
9. When will you say, a random variable is said to be centered at some constant c and its expectation?
10. State Markov's theorem.

SECTION B

Answer any **FIVE** from the following **(5x8=40)**

11. Prove that, The intersection of arbitrary number of fields is a field.
12. Explain: Induced Probability Space with an example.
13. Show that Standard Gamma variate converges in distribution to Standard Normal variate.
14. Let (X, Y) ~ BVN ($\mu_1, \mu_2, \sigma_1, \sigma_2, \rho$). Derive the marginal density function of X.
15. State and prove the properties of Expectation of Non negative random variables.
16. Prove that, If $X_n \xrightarrow{p} X$ then, there exists a subsequence $\{X_{nk}\}$ of $\{X_n\}$ which converges a.s. to X.
17. Prove that, If $\sum_1^n x_k = S_n = S < \infty$ and $b_n \uparrow \infty$, then $\frac{1}{b_n}\sum_n^1 b_k x_k \to 0$.
18. For a series of independent r.v's, prove that,
    a. Convergence in probability and in law are equivalent.
    b. If $|X_n| \le b$, for some b and E (Xn) =0 for all n then convergence in q.m. in probability and in law are equivalent.

SECTION C

Answer the following **2x20=40**

19. i) Let ξ_i be the class of all intervals of the form (a, b), (a<b) a,b Ɛ R, but arbitrary. Then P.T. $\sigma(\xi_i)$ =Ɓ. (10)

    ii) Prove that $X_n \xrightarrow{p} c$ implies that $F(X_n) \longrightarrow 0$ for x<c, $F(X_n) \longrightarrow 1$ for x≥c and conversely. (10)

(OR)

20. i) Let $X_n \xrightarrow{L} X, Y_n \xrightarrow{L} c$, then P.T.

a. $X_n + Y_n \xrightarrow{L} X + c$

b. $X_n Y_n \xrightarrow{L} cX$

c. $X_n / Y_n \xrightarrow{L} X / c$. (10)

ii) State and prove the Monotone Convergence theorem. (10)

21. i) Derive the Kolmogorov Inequality. (10)

ii) State and prove the necessary and sufficient condition for a series of random variables to converge a.s. (10)

(OR)

22. i) State and prove Liapounov's theorem. (12)

ii) Let $\{x_n\}$ be a sequence of i.i.d. r.v.'s with characteristic function φ(u). Then prove that $S_n/n \xrightarrow{p} E(X)$. (8)

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